Indian J. Phys. **71B** (4), 527–532 (1997)

NOTE

IJP B
- an international journal

New stigmatic mounting of Rowland type concave holographic diffraction grating

Patric Mulife and Shyam Singh*
Physics Department, University of Zambia, P. O Box-32379, Lusaka, Zambia

Received 16 October 1996, accepted 25 March 1997

Abstract : In this paper the focusing conditions of a new stigmatic mounting of Rowland type holographic concave diffraction grating have been obtained by applying Fermat's principle of least distance on the optical path function The design parameters which give zero astigmatism and zero coma at least at one wavelength have been presented It is found that the normal incidence grating mount gives better results than the grazing incidence grating mounting

Keywords : Holographic concave grating, stigmatic aberration, design parameters

PACS No. : 42.40.Eq

A general theory of the image formation of the concave diffraction grating was introduced for the first time by Rowland and further developed by Beutler [1] based on Fermat's principle He discussed the focussing conditions, astigmatism and coma in detail for a grating ruled conventionally. Michael [2] dealt with the aberration theory with respect to various mountings that have been designed since the advent of holographic gratings. Singh and Reddy [3–6] offered a new approach towards the design of the Seya-Namioka mounting. Noda and coworkers [7,8] and Singh [9] developed the geometric theory of a holographic concave diffraction grating and those of a mechanically ruled gratings. The major problem in most of the optical elements is the aberration in the images they produce. It is very difficult to design mechanically an optical element free from troublesome aberrations such as astigmatism and coma. Elimination of these aberrations at the same time in an optical element using mechanical methods is far more difficult.

Holographic gratings can be used both as diffracting and as focussing elements. Another advantage of holographic diffraction gratings is that they can be fabricated on

* Present address : Laser Programme, Fusion Laboratory Centre for Advanced Technology, Indore-452 013, India

arbitrary surfaces. In this case, aberrations become a function of the recording geometry and recording wavelength (r_3, r_4, α_1, α_2, λ_0). Thus by using holographic techniques, it is possible to eliminate astigmatism or coma at least for one particular wavelength of the incident light as the recording parameters can easily be controlled. Though it is a tedious job, but two aberrations are possible to balance at the same time for a particular wavelength using the holographic techniques.

In this paper, the focussing conditions of a new stigmatic mounting of Rowland type holographic concave diffraction grating have been discussed. The conditions for minimum astigmatism have been derived and discussed. The recording parameters for the case when astigmatism and coma are zero, are given.

We consider the recording set up as given earlier [9] where the centre O of the grating to be the origin of the Cartesian coordinate system with the X-axis normal at O and the Z-axis parallel to the rulings such that the tangent at their middle points are all parallel to the Z-axis. Let A (X_1, Y_1, Z_1) be a real point light source of wavelength λ on the slit and B (X_2, Y_2, Z_2) the diffracted image on the spectrum line. The projections of OA and OB onto the XY-plane from the origin are of lengths r_1 and r_2 and make the angles α_1 and α_2 respectively with the X-axis. P is a point on the grating having coordinates u, v and w corresponding to X, Y and Z-axes respectively.

We further, assume that the coherent sources of wavelength λ_0 are represented by C (X_3, Y_3, 0) and D (X_4, Y_4, 0). The angles of the recording sources *i.e.* those represented by OC and OD with the X-axis are indicated by α_3 and α_4 respectively. The optical path difference of these recording sources from the origin O is an integral multiple of λ_0. If the zero-th groove passes through the origin then the n-th groove is formed according to the equation

$$n = [\langle CP\rangle - \langle DP\rangle] - [\langle CO\rangle - \langle DO\rangle]/\lambda_0, \tag{1}$$

where $\langle\ \rangle$ indicates the optical path length. P (u, v, w) is a point of incidence on the grating for the incident ray AP while the diffracted ray is depicted by PB. The wavelength of the coherent sources has been taken to be 457.93. Now for the ray path APB, the Optical Path Function (OPF) is given by

$$F = \langle AP\rangle + \langle PB\rangle + nm\lambda, \tag{2}$$

where n is an integer and m is the order of the spectrum. Now introducing eq. (1) into eq. (2), the following expression of OPF is obtained

$$F = F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + \ldots \tag{3}$$

where

$$F_1 = r_1(1 + z_1 / r_1)^{1/2} + r_2(1 + z_2 / r_2)^{1/2} + vm\lambda / \sigma_0$$

$$- v[(1 + z_1 / r_1)^{-1/2} \sin\alpha_1 + (1 + z_2 / r_2)^{-1/2} \sin\alpha_2$$

$$F_2 = (v^2 / 2)\left[(C_1 + C_2) + (C_3 - C_4)d\right]$$

$$F_3 = (w^2/2)\left[(S_1+S_2) + (S_3-S_4)d\right]$$
$$- wz_1/r_1 - wz_2/r_2 + z_1^2/2r_1 + z_2^2/2r_2,$$
$$F_4 = \left(v^3/2\right)\left[\left\{(C_1/r_1)\sin\alpha_1 + (C_2/r_2)\sin\alpha_2\right\}\right.$$
$$+ (z_1^2-2wz_1)v\sin\alpha_1/2r_1^2 + (z_2^2-2wz_2)v\sin\alpha_2/2r_2^2$$
$$\left. + \left\{(C_3/r_3)\sin\alpha_3 - (C_4/r_4)\sin\alpha_4\right\}d\right],$$
$$F_5 = (vw^2/2)\left[\left\{(S_1/r_1)\sin\alpha_1 + (S_2/r_2)\sin\alpha_2\right\}\right.$$
$$\left.\left\{(S_3/r_3)\sin\alpha_3 - (S_4/r_4)\sin\alpha_4\right\}d\right], \tag{4}$$

where
$$C_j = (1/r_j)\cos^2\alpha_j - (1/R)\cos\alpha_j, \tag{5a}$$
$$S_j = (1/r_j) - (1/R)\cos\alpha_j, \tag{5b}$$
$$Q_1 = \sin\alpha_1 + \sin\alpha_2, \tag{5c}$$
$$Q_2 = \sin\alpha_4 - \sin\alpha_3, \tag{5d}$$
$$d = m\lambda/\lambda_0 = Q_1/Q_2 \tag{5e}$$
and
$$\sigma_0 = \lambda_0/Q_2. \tag{5f}$$

The subscript j in eqs. (5) may have a value from 1 to 4 corresponding to the points A, B, C and D respectively as given in [9]. In the expression 4, the higher order terms have been omitted because they converge rapidly for higher orders. The angles α_j are all taken to be positive if measured counter clockwise from the grating normal towards the corresponding ray. Further, the signs of α_1 and α_2 should be consistent with the signs of α_3 and α_4. The first term F_1 of eq. (3) gives the well known grating equation with the grating constant σ_0. The second term F_2 and the third term F_3 give the tangential and sagittal focussing conditions respectively. These two terms are very important terms of OPF and will be used for further developing and setting the conditions of stigmatic mountings. The term F_4 can be used to determine the useable width (Full width) of the grating while the fifth term F_5 gives the tangential and sagittal coma.

By the application of Fermat's principle *viz.* $\partial F/\partial v = 0$ and $\partial F/\partial w = 0$, and by simple derivations of OPF, the following relations are obtained :

The grating equations :

$$\partial F/\partial v = 0 \Rightarrow (\sin\alpha_1 + \sin\alpha_2) = m\lambda/\sigma_0. \tag{6}$$

The tangential focussing condition :

$$\partial F/\partial v = 0 \Rightarrow (C_1 + C_2) + d(C_3 - C_4) = 0. \tag{7}$$

The sagittal focussing condition :

$$\partial F_3/\partial w = 0 \Rightarrow (S_1 + S_2) + d(S_3 - S_4) = 0. \tag{8}$$

Eqs. (7) has nine degrees of freedom and hence, we have nine choices to satisfy this equation. However, not all the solutions can fit into the physical condition of a grating mounting. In order to eliminate the defect of focus in the image for the present case, we take $C_1 = 0$ as the source curve condition which gives

$$r_1 = R\cos\alpha_1, \tag{9}$$

a Rowland circle solution. This immediately gives the focal curve condition

$$r_2 = R\cos^2\alpha_2 / (\cos\alpha_1 + \sin\alpha_1 + \sin\alpha_2) = R\cos^2\alpha_2 / \rho, \tag{10}$$

which is also a close curve for the small values of angle of incidence making it suitable for normal incidences preferably between 0° and 10° (all the angles are measured in degrees with respect to the normal). These two eqs. (9,10) make our grating mounting different from those reported so far in the literature. In order to fit the solutions (9) and (10) and use them for the present grating mounting, we have taken the first recording condition as

$$R(C_3 - C_4) = Q_2. \tag{11}$$

The first and the most serious aberration of a grating system is astigmatism and is given by the third term F_3 of the light path function. If w is the total length of a groove projected on to the Z-axis, then the length of the astigmatic images due to a point light source can be calculated by applying Fermat's principle for w coordinate and is given by

$$Z_{ast} = Z_2 = \left[(wQ_1\cos\alpha_2)/P\right]\left[N_a - S_a\right), \tag{12}$$

where

$$N_a(\alpha_1, \alpha_2) = (\sec\alpha_1 + P\sec^2\alpha_2 - \cos\alpha_1 - \cos\alpha_2)/Q_1, \tag{13}$$

$$S_a(r_3, r_4, \alpha_3, \alpha_4) = R(S_4 - S_3)/Q_2. \tag{14}$$

It is evident from eq. (12) that astigmatism will be zero if

$$R(S_4 - S_3)/Q_2 = N_a. \tag{15}$$

Eq. (15) gives the second recording condition and generally referred as the stigmatic condition. For an arbitrary suitable value of N_a, the general equations for r_3 and r_4 for the case when astigmatism will be zero, can be obtained using eqs. (15) and (11) and are given by

$$R/r_3 = \frac{Q_2(N_a\cos^2\alpha_4 - 1) + (\cos\alpha_3 - \cos\alpha_4)\sin^2\alpha_4}{\cos^2\alpha_3 - \cos^2\alpha_4} \tag{16}$$

$$R/r_4 = \frac{Q_2(N_a\cos^2\alpha_3 - 1) + (\cos\alpha_3 - \cos\alpha_4)\sin^2\alpha_3}{\cos^2\alpha_3 - \cos^2\alpha_4} \tag{17a}$$

$$R/r_4 = R/r_3 - (\cos\alpha_3 - \cos\alpha_4) + Q_2N_a. \tag{17b}$$

The parameters r_3 and r_4 calculated from eqs. (16) and (17) for any assumed values of α_3 and α_4 for a particular value of N_a will give zero astigmatism at least at one particular wavelength. Similarly, one can obtain the conditions for zero coma. We have calculated a set of the recording parameters for both stigmatic (astigmatism corrected grating) and comatic (coma corrected grating) at normal and grazing incidences as given in Table 1.

Table 1.

Recording parameters for stigmatic mounting.
Set A : At normal incidence
$\alpha_1 = 5°$; $\alpha_2 = -3.9°$; $\alpha_3 = -18°$; $\alpha_4 = 37.356°$
$r_4 = 0.590$ m; $r_3 = 2.912$ m
Radius of curvature $R = 1$ m; Grating width = 0.0094 m
Set B : At grazing incidence
$\alpha_1 = 80°$; $\alpha_2 = -79°$; $\alpha_3 = 3°$, $\alpha_4 = 75.51°$
$r_4 = 0.003$ m; $r_3 = 0.0048$ m
Radius of curvature $R = 1$ m, Grating width = 0.0000706 m
Recording parameters for comatic grating
Set C : At normal incidence
$\alpha_1 = 5°$; $\alpha_2 = -4.1°$; $\alpha_3 = -18°$; $\alpha_4 = 20.89°$
$r_4 = 0.83$ m; $r_3 = 2.51$ m
Radius of curvature $R = 1$ m; Grating width = 0.0327 m
Set D : At grazing incidence
$\alpha_1 = 80°$, $\alpha_2 = -76°$; $\alpha_3 = -29.8°$; $\alpha_4 = 24.76°$
$r_4 = 0.035$ m; $r_3 = 0.0226$ m
Radius of curvature $R = 1$ m; Grating width = 0.0006605 m

Using holographic technique, a stigmatic grating can be designed easily to be used at a particular angle of incidence; for example, if we consider a particular angle of incidence, then the angle of diffraction can be calculated from the grating equation choosing a particular value of grating constant. Now after fixing the angle of incidence, angle of diffraction and the grating constant, one can calculate the recording lengths from eqs. (16) and (17) for the assumed values of α_3 and α_4. The grating constant for the present case has been considered to be 500 nm for the recording laser wavelength of 457.93 nm.

For each aberration, there is a separate condition to be satisfied in order to eliminate it completely. These conditions are independent and hence, when one parameter is under control for one aberration, the other one is out of control for the other aberration. It seems very difficult to eliminate all the aberrations of an optical elements produced either mechanically or holographically. However, there are more degrees of freedom available in the holographic technique and hence reducing these aberrations becomes bit easier. It is possible to balance two aberrations at the same time at least for one particular wavelength but it will be a time consuming job. Hence, we conclude that by the use of holographic technique, aberrations can be reduced to a minimum easily and a single aberration can be eliminated completely. The resolution can be increased considerably using this technique since the laser light is very sharp. In Table 1, Set A and Set B contain the recording parameters which give zero astigmatism with minimum coma at normal incidence and grazing incidence respectively. However, if we wish to eliminate coma completely then astigmatism can be adjusted to a minimum only and can not be reduced to zero at the same time. We call such a grating as coma corrected grating or comatic grating. The recording parameters for the comatic grating are given in Table 1 as Set C and Set D for normal and grazing incidences respectively.

We find that the proposed stigmatic mounting of holographic concave diffraction grating will give better results at normal incidence because the intensity of the spectrum and the resolution of the grating are both greater as compared to grazing incidence. Further, the grating width is also larger in the case of normal incidence than grazing incidence. The second best choice is the normal incidence comatic holographic concave diffraction grating although coma does not reduce to zero.

Acknowledgments

The authors thank the University of Zambia (UNZA), the Third World Academy of Sciences (TWAS), Italy, and the Centre for Advanced Technology (CAT), Indore.

References

[1] H G Beutler *J Opt. Soc Am.* **35** 311 (1945)

[2] P Michael Crisp *Applied Optics and Optical Engineering* **X** 391 (1987)

[3] Mahipal Singh and G Prakash Reddy *Optik* **74** 142 (1986)

[4] Mahipal Singh and G Prakash Reddy *Optik* **76** 83 (1987)

[5] Mahipal Singh and G Prakash Reddy *J Optics* (Paris) **19** 77 (1988)

[6] Mahipal Singh *Opt. Commun* **73** 85 (1986)

[7] H Noda, T Namioka and M Seya *J. Opt. Soc. Am.* **64** 1043 (1974)

[8] H Noda and T Namioka *J. Opt. Soc. Am.* **64** 1037 (1974)

[9] S Singh *Indian J. Pure Appl Phys.* **32** 514 (1994)